TEST OF MATHEMATICS
FOR UNIVERSITY ADMISSION

D513/11

PAPER 1

Wednesday 8 November 2017

Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.
There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

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1   Given that

$$\frac{dy}{dx} = 3x^2 - \frac{2-3x}{x^3}, \quad x \neq 0$$

and $y = 5$ when $x = 1$, find y in terms of x.

A $y = \frac{1}{3}x^3 + x^{-2} - 3x^{-1} + 6\frac{2}{3}$

B $y = x^3 + \frac{1}{2}x^{-2} - 3x^{-1} + 6\frac{1}{2}$

C $y = x^3 + x^{-2} - 3x^{-1} + 6$

D $y = x^3 + x^{-2} - x^{-1} + 4$

E $y = x^3 + 2x^{-2} - x^{-1} + 3$

F $y = 3x^3 + x^{-2} - x^{-1} + 2$

2 The function f is given by

$$f(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2 \quad (x \neq 0)$$

What is the value of $f''(1)$?

A -3

B -1

C 5

D 17

E 29

F 80

3 A line l has equation $y = 6 - 2x$

A second line is perpendicular to l and passes through the point $(-6, 0)$.

Find the area of the region enclosed by the two lines and the x-axis.

A $16\dfrac{1}{5}$

B 18

C $21\dfrac{3}{5}$

D 27

E $40\dfrac{1}{2}$

When $(3x^2 + 8x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$, the remainder is 24.

What is the value of p?

A -4

B 2

C 4

D $\dfrac{8}{7}$

E $\dfrac{11}{4}$

5 S is the complete set of values of x which satisfy **both** the inequalities

$$x^2 - 8x + 12 < 0 \text{ and } 2x + 1 > 9$$

The set S can also be represented as a single inequality.

Which one of the following single inequalities represents the set S ?

A $(x^2 - 8x + 12)(2x + 1) < 0$

B $(x^2 - 8x + 12)(2x + 1) > 0$

C $x^2 - 10x + 24 < 0$

D $x^2 - 10x + 24 > 0$

E $x^2 - 6x + 8 < 0$

F $x^2 - 6x + 8 > 0$

G $x < 2$

H $x > 6$

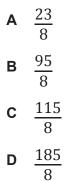6   A tangent to the circle $x^2 + y^2 = 144$ passes through the point $(20, 0)$ and crosses the positive y-axis.

What is the value of y at the point where the tangent meets the y-axis?

    A   12

    B   15

    C $\dfrac{49}{3}$

    D   20

    E $\dfrac{64}{3}$

    F $\dfrac{80}{3}$

7 The first three terms of an arithmetic progression are p, q and p^2 respectively, where $p < 0$

The first three terms of a geometric progression are p, p^2 and q respectively.

Find the sum of the first 10 terms of the arithmetic progression.

A $\dfrac{23}{8}$

B $\dfrac{95}{8}$

C $\dfrac{115}{8}$

D $\dfrac{185}{8}$

8 Find the complete set of values of x, with $0 \leq x \leq \pi$, for which

$$(1 - 2 \sin x) \cos x \geq 0$$

A $0 \leq x \leq \dfrac{\pi}{6}, \quad \dfrac{\pi}{2} \leq x \leq \dfrac{5\pi}{6}$

B $0 \leq x \leq \dfrac{\pi}{6}, \quad \dfrac{5\pi}{6} \leq x \leq \pi$

C $\dfrac{\pi}{6} \leq x \leq \dfrac{\pi}{2}, \quad \dfrac{5\pi}{6} \leq x \leq \pi$

D $\dfrac{\pi}{6} \leq x \leq \dfrac{5\pi}{6}$

9   A circle has equation $x^2 + y^2 - 18x - 22y + 178 = 0$

A regular hexagon is drawn inside this circle so that the vertices of the hexagon touch the circle.

What is the area of the hexagon?

A 6

B $6\sqrt{3}$

C 18

D $18\sqrt{3}$

E 36

F $36\sqrt{3}$
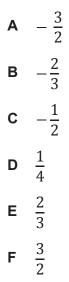
G 48

H $48\sqrt{3}$

10   A curve C has equation $y = f(x)$ where

$$f(x) = p^3 - 6p^2x + 3px^2 - x^3$$

and p is real.

The gradient of the normal to the curve C at the point where $x = -1$ is M.

What is the greatest possible value of M as p varies?

A $-\dfrac{3}{2}$

B $-\dfrac{2}{3}$

C $-\dfrac{1}{2}$

D $\dfrac{1}{4}$

E $\dfrac{2}{3}$

F $\dfrac{3}{2}$

11 The sequence x_n is defined by the rules

$$x_1 = 7$$

$$x_{n+1} = \frac{23x_n - 53}{5x_n + 1}$$

The first three terms in the sequence are 7, 3, 1

What is the value of x_{100}?

A -5

B 0

C 1

D 3

E 7

12 The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of x.

Given $\int_2^4 f(x)\,dx = A$, which one of the following statements **must** be correct?

A $\quad \displaystyle\int_0^2 [f(x+2)+1]\,dx = A+1$

B $\quad \displaystyle\int_0^2 [f(x+2)+1]\,dx = A+2$

C $\quad \displaystyle\int_2^4 [f(x+2)+1]\,dx = A+1$

D $\quad \displaystyle\int_2^4 [f(x+2)+1]\,dx = A+2$

E $\quad \displaystyle\int_4^6 [f(x+2)+1]\,dx = A+1$

F $\quad \displaystyle\int_4^6 [f(x+2)+1]\,dx = A+2$

13 In the expansion of $(a + bx)^5$ the coefficient of x^4 is 8 times the coefficient of x^2.

Given that a and b are non-zero **positive** integers, what is the smallest possible value of $a + b$?

A 3

B 4

C 5

D 9

E 13

F 17

14 The solution of the simultaneous equations

$$2^x + 3 \times 2^y = 3$$

$$2^{2x} - 9 \times 2^{2y} = 6$$

is $x = p,\ y = q.$

Find the value of $p - q$

A $\dfrac{5}{12}$

B $\dfrac{7}{3}$

C $\log_2 \dfrac{5}{12}$

D $\log_2 \dfrac{7}{3}$

E $\log_2 9$

F $\log_2 15$

15 It is given that $f(x) = -2x^2 + 10$

Consider the following three curves:

> $(\mathbf{1})\ y = f(x)$
>
> $(\mathbf{2})\ y = f(x+1)$
>
> $(\mathbf{3})$ the curve $y = f(x+1)$ reflected in the line $y = 6$

The trapezium rule is used to estimate the area under each of these three curves between $x = 0$ and $x = 1$.

State whether the trapezium rule gives an overestimate or underestimate for each of these areas.

	(**1**)	(**2**)	(**3**)
A	underestimate	underestimate	underestimate
B	underestimate	underestimate	overestimate
C	underestimate	overestimate	underestimate
D	underestimate	overestimate	overestimate
E	overestimate	underestimate	underestimate
F	overestimate	underestimate	overestimate
G	overestimate	overestimate	underestimate
H	overestimate	overestimate	overestimate

16 The functions f and g are given by $f(x) = 3x^2 + 12x + 4$ and $g(x) = x^3 + 6x^2 + 9x - 8$.

What is the complete set of values of x for which one of the functions is increasing and the other decreasing?

A $x \geq -1$

B $x \leq -1$

C $-3 \leq x \leq -2, \ x \geq -1$

D $x \leq -2, \ x \geq -1$

E $x \leq -3, \ -2 \leq x \leq -1$

F $x \leq -3, \ x \geq -2$

G $-2 \leq x \leq -1$

17 The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers n:

$$F(n) = \frac{1}{n}\int_0^n (n-x)\,dx$$

$$G(n) = \sum_{r=1}^n F(r)$$

What is the smallest positive integer n such that $G(n) > 150$?

A 22

B 23

C 24

D 25

E 26

18 The graph of $y = \log_{10} x$ is translated in the positive y-direction by 2 units.

This translation is equivalent to a stretch of factor k parallel to the x-axis.

What is the value of k?

A 0.01

B $\log_{10} 2$

C 0.5

D 2

E $\log_2 10$

F 100

19 The set of solutions to the inequality $x^2 + bx + c < 0$ is the interval $p < x < q$

where b, c, p and q are real constants with $c < 0$.

In terms of p, q and c, what is the set of solutions to the inequality $x^2 + bcx + c^3 < 0$?

A $\dfrac{p}{c} < x < \dfrac{q}{c}$

B $\dfrac{q}{c} < x < \dfrac{p}{c}$

C $pc < x < qc$

D $qc < x < pc$

E $pc^2 < x < qc^2$

F $qc^2 < x < pc^2$

20 The lengths of the sides QR, RP and PQ in triangle PQR are a, $a + d$ and $a + 2d$ respectively, where a and d are positive and such that $3d > 2a$.

What is the full range, in degrees, of possible values for angle PRQ?

A $\quad 0 < \text{angle } PRQ < 60$

B $\quad 0 < \text{angle } PRQ < 120$

C $\quad 60 < \text{angle } PRQ < 120$

D $\quad 60 < \text{angle } PRQ < 180$

E $\quad 120 < \text{angle } PRQ < 180$

END OF TEST

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