Test of Mathematics for University Admission, 2019 Paper 1 Worked Solutions

Contents

Introduction for students

These solutions are designed to support you as you prepare to take the Test of Mathematics for University Admission. They are intended to help you understand how to answer the questions, and therefore you are strongly encouraged to **attempt the questions first** before looking at these worked solutions. For this reason, each solution starts on a new page, so that you can avoid looking ahead.

The solutions contain much more detail and explanation than you would need to write in the test itself – after all, the test is multiple choice, so no written solutions are needed, and you may be very fluent at some of the steps spelled out here. Nevertheless, doing too much in your head might lead to making unnecessary mistakes, so a healthy balance is a good target!

There may be alternative ways to correctly answer these questions; these are not meant to be 'definitive' solutions.

The questions themselves are available on the 'Preparing for the test' section on the Admissions Testing website.

Question 1

We could try substituting in $(1, -1)$ and $(-1, 3)$ into the six given expressions: when we substitute in $x = 1$, only A, C, D and F give -1, so the answer must be one of these. When we substitute in $x = -1$ into these four, only A and C give 3, so the answer must be one of these two. Finally, we need a turning point at $x = -1$, so we differentiate these two: A gives $-2x-2$, which is zero at $x = -1$, which C gives $2x - 2$, which is -4 at $x = -1$. So the correct answer is A.

Alternatively, we could work out the answer without reference to the given options.

Let $\mathrm{f}(x) = ax^2 + bx + c$. We are given $\mathrm{f}(1) = -1$ and $\mathrm{f}(-1) = 3$, so

$$\begin{aligned} a + b + c &= -1 \\ a - b + c &= 3 \end{aligned}$$

Adding these gives $2a + 2c = 2$, so $a + c = 1$ and hence $b = -2$.

We then differentiate to get $\mathrm{f}'(x) = 2ax + b$, so as $\mathrm{f}'(-1) = 0$, we require $-2a + b = 0$, so $a = -1$ and $c = 2$. Hence the answer is A.

Question 2

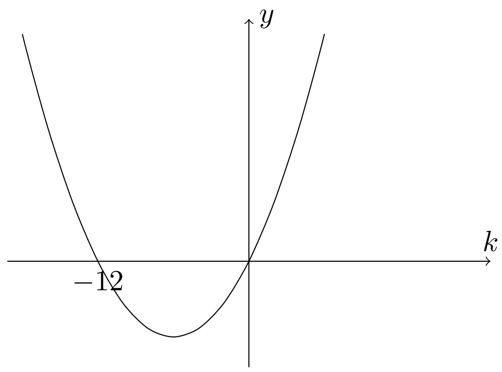
Let us write the expression as $x^2+(k+2)x+(1-2k)$ so that the coefficient of x and the constant are clear. The expression is positive for large values of x, and so is positive for all values of x if the discriminant is negative. Hence we require

$$(k+2)^2 - 4(1-2k) < 0.$$

Expanding and simplifying this gives

$$k^2 + 12k < 0,$$

so $k(k+12) < 0$. If we sketch a graph of the function $k(k+12)$, we obtain



We see that $k(k+12) < 0$ for $-12 < k < 0$, so the correct option is A.

Question 3

The coefficient of x in the expansion of $(1+x)^n$ is $\binom{n}{1} = n$, so the coefficient of x in this whole expression is

$$0 + 1 + 2 + 3 + \cdots + 79 + 80 = \tfrac{1}{2} \times 80 \times 81$$

using the formula for the sum of the first n positive integers, $\frac{1}{2}n(n+1)$. This gives $40 \times 81 = 3240$, so the answer is E.

Question 4

We work out the first few terms, writing them in the form of the given options:

$$\begin{aligned}
x_1 &= 10 \\
x_2 &= \sqrt{10} = 10^{\frac{1}{2}} = 10^{2^{-1}} \\
x_3 &= \sqrt{10^{2^{-1}}} = \left(10^{2^{-1}}\right)^{\frac{1}{2}} = 10^{2^{-1} \times \frac{1}{2}} = 10^{2^{-2}}
\end{aligned}$$

As this pattern continues, we will end up with $x_{100} = 10^{2^{-99}}$, which is option C.

Question 5

Let the first term of S be a and the common ratio r, as usual.

The sum of the first n terms of S is $\dfrac{a(r^n - 1)}{r - 1}$, so the second line of the question can be written as

$$\frac{a(r^6 - 1)}{r - 1} = \frac{9a(r^3 - 1)}{r - 1}$$

which can be multiplied by $(r - 1)$ and divided by a to give

$$r^6 - 1 = 9(r^3 - 1).$$

Writing $R = r^3$, this becomes $R^2 - 1 = 9R - 9$, so $R^2 - 9R + 8 = 0$. This factorises to $(R - 1)(R - 8) = 0$, so $R = 1$ or $R = 8$, and hence $r = 1$ or $r = 2$.

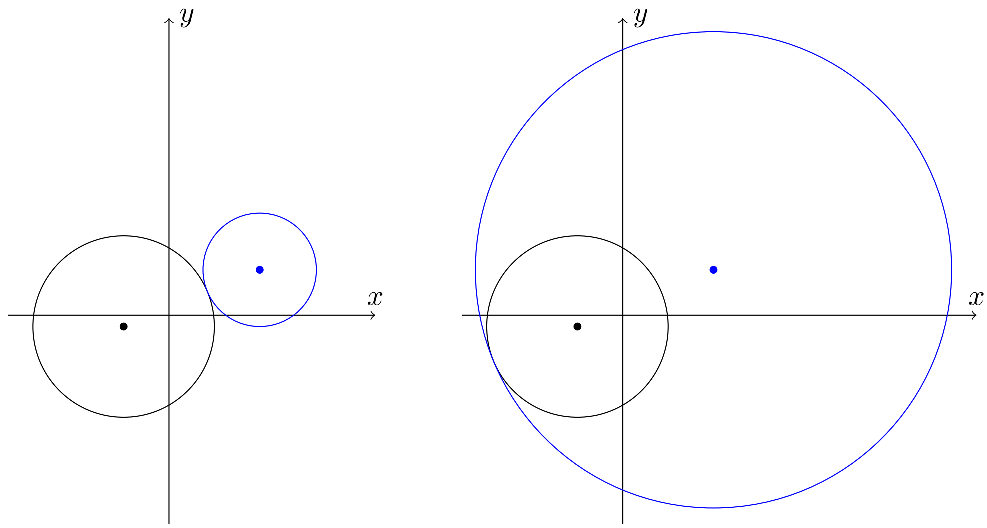
However, if $r = 1$, then the original equation is not valid; in that case, the sequence is constant, the sum of the first 6 terms is $6a$ and the sum of the first 3 terms is $3a$. We could only have $6a = 9 \times 3a$ if $a = 0$, but that is impossible as the 7th term of S is 360. So we must have $r = 2$.

The 7th term of S is ar^6, so $360 = 2^6 a = 64a$, and hence $a = \frac{360}{64} = \frac{45}{8}$ and so the answer is E.

Question 6

We could try to solve this algebraically or geometrically. An algebraic approach looks difficult, as we have two quadratic equations, and one of them has an unknown coefficient (that is, r^2). So we use a geometric approach instead.

The first circle has centre $(-4, -1)$ and radius 8, while the second circle has centre $(8, 4)$ and radius r. The point $(8, 4)$ does not lie inside the first circle (it is more than $8 - (-4) = 12$ units away), so the two circles will have exactly one point in common when the first circle is tangent to the second circle externally or when it lies inside the second circle and touches it internally. Here is a sketch of these two situations:



In the first case, where $r = r_1$, we see that $r_1 + 8$ is the distance between the centres of the two circles. In the second case, where $r = r_2$, we see that $r_2 - 8$ is the distance between the centres.

Therefore $r_1 + 8 = r_2 - 8$, so $r_2 - r_1 = 16$, and the answer is C.

Note that we did not even need to work out the actual values of r_1 and r_2. If we had done so, the distance between the centres is $\sqrt{12^2 + 5^2} = 13$, so $r_1 = 13 - 8 = 5$ and $r_2 = 13 + 8 = 21$.

Question 7

We expand the given expression to get

$$y = 4q^2x - 2qx^3 + 6q - 3x^2$$

Differentiating gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4q^2 - 6qx^2 - 6x$$

so the gradient of the curve at $x = -1$ is $4q^2 - 6q + 6$. We can minimise this either by using calculus or by completing the square.

Using calculus, we have $\frac{\mathrm{d}}{\mathrm{d}q}(4q^2 - 6q + 6) = 8q - 6$, and $8q - 6 = 0$ when $q = \frac{6}{8} = \frac{3}{4}$, so the answer is F.

Alternatively, completing the square, we have

$$4q^2 - 6q + 6 = 4(q^2 - \tfrac{3}{2}q) + 6 = 4\big((q - \tfrac{3}{4})^2 - (\tfrac{3}{4})^2\big) + 6$$

so the minimum occurs when $q = \frac{3}{4}$, as before.

Question 8

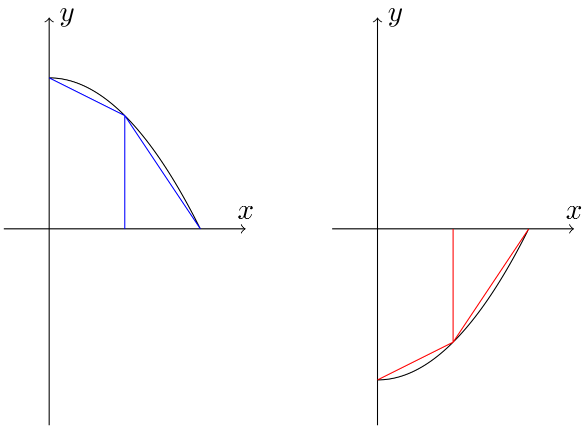
A This translates the graph upwards (in the positive y-direction) by 1 unit, so the trapezia will just be 1 unit taller. The approximation will therefore still be an underestimate.

B This stretches the graph by a factor of 2 in the y-direction, and the trapezia with it, so the approximation will still be an underestimate.

C This translates the graph by 1 unit leftwards (in the negative x-direction), and correspondingly calculates the area between -1 and 0 rather than between 0 and 1; the trapezia are correspondingly translated leftwards by 1 unit, and so we will still get an underestimate.

D This reflects the graph in the y-axis, and correspondingly calculates the area between -1 and 0. The trapezia are correspondingly reflected, so we will still get an underestimate.

E The graph is reflected in the x-axis and then translated upwards by 1 unit. Whether the approximation using the trapezium rule is an over- or underestimate is not affected by the vertical translation, as we noted in **A**, so we may as well not translate the graph at all, giving just $\int_0^1 (-\mathrm{f}(x))\,\mathrm{d}x$. Let us sketch an example using just two intervals:

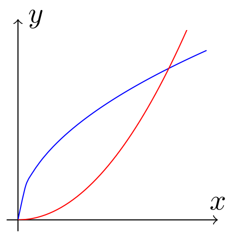


The original (blue) trapezia give an underestimate for the integral. The red trapezia give an underestimate for the *area* of the region, but since the integral is negative (the graph lies below the x-axis), the trapezium rule gives an overestimate for the *integral* (for example, it may estimate -0.6 instead of the correct -0.65, and $-0.6 > -0.65$).

Hence the correct answer is E.

Question 9

We can rearrange $x = p\sqrt{y}$ to give $\sqrt{y} = \dfrac{x}{p}$, so $y = \dfrac{x^2}{p^2}$ (for $x \geq 0$). We can then sketch the two curves to see what is intended:



We need to find where these two curves intersect. We have

$$p\sqrt{x} = \frac{x^2}{p^2}$$

so $x^{\frac{3}{2}} = p^3$, giving $x^{\frac{1}{2}} = p$, so $x = p^2$ and thus $y = p^2$ as well; the point of intersection thus lies on the line $y = x$ and the graph is symmetrical about this line.

We can now find the area between the curves by integrating the difference from $x = 0$ to $x = p^2$:

$$\begin{aligned}
\text{area} &= \int_0^{p^2} p\sqrt{x} - \frac{x^2}{p^2}\,\mathrm{d}x \\
&= \left[\frac{px^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{x^3}{3p^2}\right]_0^{p^2} \\
&= \frac{p.p^3}{\left(\frac{3}{2}\right)} - \frac{p^6}{3p^2} - 0 \\
&= \tfrac{2}{3}p^4 - \tfrac{1}{3}p^4 \\
&= \tfrac{1}{3}p^4
\end{aligned}$$

so the answer is D.

Question 10

We do not have a rule for integrating $|x|$, so we will split this into two separate integrals, one from $x = -1$ to $x = 0$, where $|x| = -x$, and one from $x = 0$ to $x = 3$, where $|x| = x$. This gives

$$\begin{aligned}
\int_{-1}^{3} |x|(1-x)\,\mathrm{d}x &= \int_{-1}^{0} |x|(1-x)\,\mathrm{d}x + \int_{0}^{3} |x|(1-x)\,\mathrm{d}x \\
&= \int_{-1}^{0} (-x)(1-x)\,\mathrm{d}x + \int_{0}^{3} x(1-x)\,\mathrm{d}x \\
&= \int_{-1}^{0} -x + x^2\,\mathrm{d}x + \int_{0}^{3} x - x^2\,\mathrm{d}x \\
&= \left[-\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{3} \\
&= -(-\tfrac{1}{2} - \tfrac{1}{3}) + (\tfrac{9}{2} - 9) \\
&= \tfrac{5}{6} - \tfrac{9}{2} \\
&= -\tfrac{22}{6} \\
&= -\tfrac{11}{3}
\end{aligned}$$

so the correct answer is F.

Question 11

We can write the first equation as $\log_3 x + 2\log_3 y = 1$. Then writing $X = \log_3 x$ and $Y = \log_3 y$, the two equations become

$$\begin{aligned} X + 2Y &= 1 \\ XY &= -3 \end{aligned}$$

We are interested in the values of X, so we rearrange the second equation to get $Y = -3/X$, and substitute this into the first equation to give

$$X - \frac{6}{X} = 1.$$

Multiplying this by X gives the quadratic $X^2 - 6 = X$, or $X^2 - X - 6 = 0$. This factorises to $(X-3)(X+2) = 0$, so $X = 3$ or $X = -2$, so $x = 3^X$ is either 27 or $\frac{1}{9}$, and their sum is $27\frac{1}{9}$, which is option H.

We could also have solved this problem by writing Y in terms of X using the first equation, and then substituting this into the second; it would have given the same result.

Question 12

To find the value of V when $t = 9$, we need to integrate this expression. There is no general rule for integrating quotients such as these, but we might notice that $t - 1$ looks somewhat similar to the denominator $\sqrt{t} + 1$. In fact, we can use the difference of two squares to write $t - 1 = (\sqrt{t} + 1)(\sqrt{t} - 1)$, so we can rewrite the fraction to give

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 24\pi(\sqrt{t} - 1)$$

and this is easy to integrate; we get

$$\begin{aligned} V &= \frac{24\pi t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 24\pi t + c \\ &= 16\pi t^{\frac{3}{2}} - 24\pi t + c \end{aligned}$$

Substituting in $V = 7$ and $t = 1$ gives

$$7 = 16\pi - 24\pi + c$$

so $c = 8\pi + 7$. Then at $t = 9$, we have

$$V = 16\pi \times 9^{\frac{3}{2}} - 24\pi \times 9 + 8\pi + 7 = (16 \times 27 - 24 \times 9 + 8)\pi + 7 = 224\pi + 7$$

so the answer is option C.

> ***Commentary:*** *The final calculation can be simplified somewhat by writing* $16 \times 27 = 16 \times 3 \times 9 = 48 \times 9$, *so the brackets become* $24 \times 9 + 8$. *Then* $24 \times 9 = 240 - 24 = 216$, *giving the final answer of* $224\pi + 7$.

Question 13

We will not use calculus for this question, as the specification does not expect knowledge of differentiating exponentials or trigonometric functions. (And it also turns out that using calculus would make the solution far more complicated.)

Instead, we note that this is a quadratic expression in $u = 2^{\sin x}$, and we can write the function as

$$u^2 - 4u + \tfrac{17}{4}.$$

We also note that as $-1 \leq \sin x \leq 1$, $\frac{1}{2} \leq u \leq 2$ for all real x (and u can take every value in this interval).

Now we can complete the square on the quadratic to understand its behaviour; we get

$$(u-2)^2 - 2^2 + \tfrac{17}{4} = (u-2)^2 + \tfrac{1}{4}$$

Thus the minimum value of the function occurs when $u = 2$, and for u in the interval $\frac{1}{2} \leq u \leq 2$, the maximum value occurs when $u = \frac{1}{2}$. In this case, we get $(-\frac{3}{2})^2 + \frac{1}{4} = \frac{10}{4} = \frac{5}{2}$, so the correct answer is B.

Question 14

We write $S = \sin 2x$ and $C = \cos 2x$, so the simultaneous equations become

$$\begin{aligned} S + \sqrt{3}C &= -1 \\ \sqrt{3}S - \quad C &= \sqrt{3} \end{aligned}$$

Multiplying the second equation by $\sqrt{3}$ and then adding to the first equation gives

$$4S = 2$$

so $S = \frac{1}{2}$ and hence $C = -\frac{\sqrt{3}}{2}$.

The solutions to $\sin 2x = \frac{1}{2}$ for $0° \leq x \leq 360°$, or $0° \leq 2x \leq 720°$, are given by $2x = 30°$, $150°$, $390°$ and $510°$.

The solutions to $\cos 2x = -\frac{\sqrt{3}}{2}$ in the same range are given by $2x = 150°$, $210°$, $510°$ and $570°$, so the values of $2x$ for which both are true are $150°$ and $510°$.

Thus the solutions for x in this range are $75°$ and $255°$, and their sum is $330°$. Hence the correct answer is option B.

Question 15

We look to see how we can simplify this problem. First, the powers are 9^x and 3^x. We can write $9^x = (3^2)^x = 3^{2x} = (3^x)^2$, but it is not clear how this will help. But without a better idea, let us go with it, and write $u = 3^x$, so the question becomes

$$\frac{2^{(u^2)}}{8^u} = \frac{1}{4}.$$

Next, let us write everything as powers of 2, giving

$$\frac{2^{(u^2)}}{2^{3u}} = \frac{1}{2^2}.$$

Therefore

$$2^{u^2-3u} = 2^{-2}$$

so we require

$$u^2 - 3u = -2.$$

We can rearrange this quadratic to give $u^2 - 3u + 2 = 0$, so $(u-1)(u-2) = 0$ giving $u = 1$ or $u = 2$. Thus $3^x = 1$ or $3^x = 2$, giving two possibilities: $x = 0$ or $x = \log_3 2$.

The question explicitly says "find the real **non-zero** solution", so the correct answer is option A.

Question 16

Let $A = \int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, which is the (signed) area under the graph of $y = \mathrm{f}(x)$ between $x = 0$ and $x = 1$ (taking signs into account, as usual: parts of the graph under the x-axis are counted as negative areas). Likewise, let $B = \int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, which is the (signed) area between $x = 1$ and $x = 2$.

The integral $\int_0^1 \mathrm{f}(x+1)\,\mathrm{d}x$ gives the (signed) area under the graph of $y = \mathrm{f}(x+1)$ between $x = 0$ and $x = 1$. This graph is the graph of $y = \mathrm{f}(x)$ translated left by 1 unit, so this equals the area under the original graph between $x = 1$ and $x = 2$, which is B. Hence $B = 6$.

The first equation gives $2A + 5B = 14$, and as $B = 6$, we get $A = -8$.

The final integral $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$ is the (signed) area between $x = 0$ and $x = 2$, which is $A + B = -8 + 6 = -2$.

Hence the answer is C.

Question 17

A product of two terms is at least 0 if both factors are positive or both terms are negative. So we look at the signs of the two factors.

We start by finding the values of θ for which they are zero.

We have $\sin 2\theta = \frac{1}{2}$ when $2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, so for $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$. Thinking about a sketch of the graph of $y = \sin 2\theta$, we see that $\sin 2\theta - \frac{1}{2}$ is positive for θ between these values and negative elsewhere.

For the second factor, we have $\sin\theta = \cos\theta$ when $\tan\theta = 1$, so $\theta = \frac{\pi}{4}$. Thinking about the graphs of $y = \sin\theta$ and $y = \cos\theta$, we see that $\sin\theta - \cos\theta$ is negative for $\theta < \frac{\pi}{4}$ and positive for $\theta > \frac{\pi}{4}$. For consistency, we write $\frac{\pi}{4} = \frac{3\pi}{12}$ in what follows.

We thus have the following table:

	$0 \le \theta < \frac{\pi}{12}$	$\theta = \frac{\pi}{12}$	$\frac{\pi}{12} < \theta < \frac{3\pi}{12}$	$\theta = \frac{3\pi}{12}$	$\frac{3\pi}{12} < \theta < \frac{5\pi}{12}$	$\theta = \frac{5\pi}{12}$	$\frac{5\pi}{12} < \theta \le \pi$
$\sin 2\theta - \frac{1}{2}$	$-$	0	$+$	$+$	$+$	0	$-$
$\sin\theta - \cos\theta$	$-$	$-$	$-$	0	$+$	$+$	$+$
product	$+$	0	$-$	0	$+$	0	$-$

The lengths of the subintervals where the product is positive (or zero) is

$$\left(\tfrac{\pi}{12} - 0\right) + \left(\tfrac{5\pi}{12} - \tfrac{3\pi}{12}\right) = \tfrac{3\pi}{12} = \tfrac{\pi}{4}$$

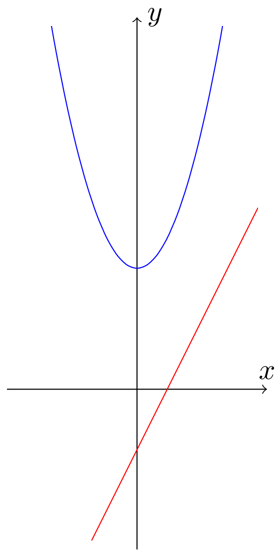
The whole interval has length π, so the fraction of the interval for which the inequality is satisfied is $\frac{1}{4}$, and the answer is option C.

Question 18

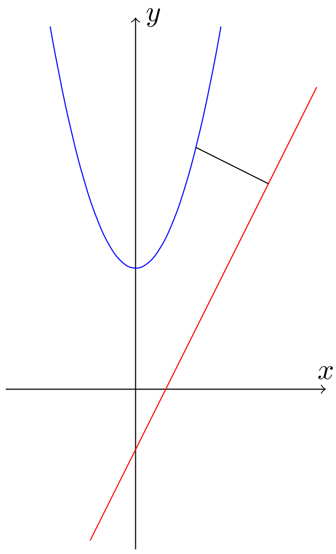
From the given options, it is clear that the curve and line do not intersect.

One way to do this would be to take a point on the curve, say (a, a^2+4), and a point on the line, say $(b, 2b-2)$, find the (square of) the distance between them, and then try to find some way of minimising it. That, though, seems quite challenging, as there are two independent variables involved.

Another way is to draw a sketch of the situation and to think about what the question means geometrically.



The shortest distance from any point on the curve to the straight line is found by drawing a perpendicular from the curve to the straight line, like this:



This may give us a few different ideas; any of the following would work:

- We can use this to find the distance of any point (a, a^2+4) to the line; we then find the smallest possible such distance.

- The smallest distance will be when the perpendicular to the line is also perpendicular to the curve; in the above sketch, it is clear that if we move the joining line closer to the origin, its length will become shorter. We can find this position by finding the gradient of the normal to the curve and finding a point at which this is perpendicular to the line.

- Taking this one step further, if we translate the line (without changing its gradient), we shorten the minimum distance of the curve from the line, but as long as we don't cross the curve, the point which is closest to the line will remain so. Therefore the closest point to the line is the point on the curve where the curve's gradient equals the line's gradient, for when the line just touches the curve, this point will be at zero distance from the curve.

We will use the final approach to solve this problem.

The gradient of the line is 2, and the curve has gradient $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$, so $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ when $x = 1$, so $(1, 5)$ is the closest point on the curve to the line.

The line through $(1, 5)$ perpendicular to the line $y = 2x - 2$ has equation $y - 5 = -\frac{1}{2}(x - 1)$. This intersects the line $y = 2x - 2$ when

$$(2x - 2) - 5 = -\tfrac{1}{2}(x - 1)$$

so $2x - 7 = -\frac{1}{2}(x - 1)$. Multiplying this by 2 gives

$$4x - 14 = -x + 1$$

so $5x = 15$, giving $x = 3$, $y = 4$.

The distance between $(1, 5)$ and $(3, 4)$ is $\sqrt{(3-1)^2 + (4-5)^2} = \sqrt{5}$, so the correct answer is B.

> ***Commentary:*** *Though we dismissed the original two-variable idea, it turns out that we could have used it in a similar way to the first idea listed above: if we fix a value for a, we can find the value of b which gives the smallest distance from that point to the line. This gives us a formula for the smallest distance from a given point to the line in terms of a; we can then find the value of a which makes this as small as possible. This changes the problem from one two-variable problem to two one-variable problems, which we know how to solve.*

Question 19

We start by writing out the first few terms of the summation, and writing each of them in terms of $\sin 10°$ or $\sin 100°$:

$$\begin{aligned}
k = 0 &: \quad \sin 10^\circ \\
k = 1 &: \quad \sin 100^\circ \\
k = 2 &: \quad \sin 190^\circ = -\sin 10^\circ \\
k = 3 &: \quad \sin 280^\circ = -\sin 100^\circ \\
k = 4 &: \quad \sin 370^\circ = \sin 10^\circ \\
k = 5 &: \quad \sin 460^\circ = \sin 100^\circ \\
k = 6 &: \quad \sin 550^\circ = -\sin 10^\circ \\
k = 7 &: \quad \sin 640^\circ = -\sin 100^\circ
\end{aligned}$$

and we see that we repeat every four terms. The first four terms sum to zero, as do the next four, and so on.

In total, there are 91 terms, and $91 = 22 \times 4 + 3$, so the only terms which contribute to the final sum are the last three. Therefore the whole sum is that from $k = 88$, $k = 89$ and $k = 90$, giving

$$\sin 10^\circ + \sin 100^\circ + (-\sin 10^\circ) = \sin 100^\circ.$$

Hence the answer is C.

Question 20

Finding the values of x at which the two curves intersect is the same as finding the values of x at which the y-values are equal, that is,

$$x^3 - 12x = k - (x-2)^2$$

or

$$x^3 - 12x - k + (x-2)^2 = 0.$$

Thus we are finding out something about the roots of a cubic.

We can expand the brackets in the final equation to give

$$x^3 + x^2 - 16x - k + 4 = 0,$$

and we write $\mathrm{f}(x) = x^3 + x^2 - 16x - k + 4$.

We can determine something about the locations of the roots of $\mathrm{f}(x)$ by finding the stationary points. We have

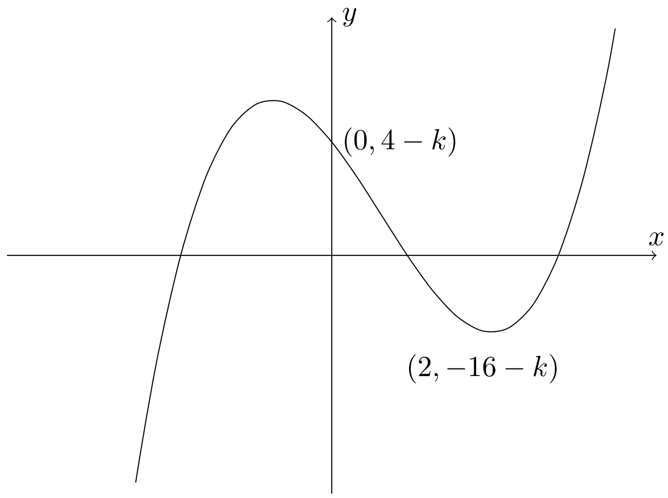
$$\mathrm{f}'(x) = 3x^2 + 2x - 16,$$

so $\mathrm{f}'(x) = 0$ when

$$x = \frac{-2 \pm \sqrt{2^2 + 4 \times 3 \times 16}}{6} = \frac{-2 \pm \sqrt{196}}{6} = \frac{-2 \pm 14}{6} = 2 \text{ or } -\frac{8}{3}.$$

(We could have found these roots by factorising, but that did not look particularly easy.) Therefore there are stationary points at $x = 2$ and $x = -\frac{8}{3}$. We have $\mathrm{f}(2) = -16 - k$, but calculating $\mathrm{f}(-\frac{8}{3})$ by hand would be painful, so let us try to avoid doing so if we can!

We note that $\mathrm{f}(0) = 4 - k$, and that $\mathrm{f}(x)$ gets big as x gets big, so if there are exactly two roots with positive x-coordinates, we need the graph to look something like this:



In particular, we require $\mathrm{f}(0) > 0$, so we must have $k < 4$.

It must also be the case that the stationary point at $x = 2$ lies below the x-axis, so we need $-16 - k < 0$, that is, $-16 < k$.

Once we have met these two conditions, it becomes clear that there are three distinct roots, exactly one of which has $x < 0$. Therefore, the complete range of values of k is given by $-16 < k < 4$, which is option E.