TMUA/CTMUA **D513/01**

PAPER 1

November 2020 **75 minutes**

Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.

PV2

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1 Which of the following is an expression for the first derivative with respect to x of

$$\frac{x^3 - 5x^2}{2x\sqrt{x}}$$

A $\quad -\dfrac{\sqrt{x}}{2}$

B $\quad \dfrac{\sqrt{x}}{4}$

C $\quad \dfrac{3x - 5}{4\sqrt{x}}$

D $\quad \dfrac{3\sqrt{x} - 5}{4\sqrt{x}}$

E $\quad \dfrac{3\sqrt{x} - 10}{3\sqrt{x}}$

F $\quad \dfrac{3x^2 - 10x}{3\sqrt{x}}$

2 $(2x + 1)$ and $(x - 2)$ are factors of $2x^3 + px^2 + q$

What is the value of $2p + q$?

A -10

B $-\frac{38}{5}$

C $-\frac{22}{3}$

D $\frac{22}{3}$

E $\frac{38}{5}$

F 10

3 Find the complete set of values of x for which

$$(x+4)(x+3)(1-x) > 0 \quad \text{and} \quad (x+2)(x-2) < 0$$

A $\quad 1 < x < 2$

B $\quad -2 < x < 1$

C $\quad -2 < x < 2$

D $\quad x < -2 \ \text{ or } \ x > 1$

E $\quad x < -4 \ \text{ or } \ x > 2$

F $\quad x < -4 \ \text{ or } \ -3 < x < 1$

G $\quad -4 < x < -2 \ \text{ or } \ x > 1$

4 The 1$^{\text{st}}$, 2$^{\text{nd}}$ and 3$^{\text{rd}}$ terms of a geometric progression are also the 1$^{\text{st}}$, 4$^{\text{th}}$ and 6$^{\text{th}}$ terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 12.

Find the 1$^{\text{st}}$ term of the geometric progression.

A 1

B 2

C 3

D 4

E 5

F 6

5 The curve S has equation

$$y = px^2 + 6x - q$$

where p and q are constants.

S has a line of symmetry at $x = -\frac{1}{4}$ and touches the x-axis at exactly one point.

What is the value of $p + 8q$?

A 6

B 18

C 21

D 25

E 38

6 Find the maximum value of the function

$$f(x) = \frac{1}{5^{2x} - 4(5^x) + 7}$$

A $\frac{1}{7}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D 3

E 4

F 7

7 Given that

$$2^{3x} = 8^{(y+3)}$$

and

$$4^{(x+1)} = \frac{16^{(y+1)}}{8^{(y+3)}}$$

what is the value of $x + y$?

A -23

B -22

C -15

D -14

E -11

F -10

8 The function f is defined for all real x as

$$f(x) = (p - x)(x + 2)$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

A $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

B $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

C $-2\sqrt{5} < p < 2\sqrt{5}$

D $-6 < p < 2$

E $-4 < p < 0$

F $-2 < p < 2$

10

9 The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

A $\quad x^2 - \sqrt{10}x + 3$

B $\quad x^2 - \sqrt{14}x + 3$

C $\quad x^2 - \sqrt{20}x + 3$

D $\quad x^2 - 178x + 81$

E $\quad x^2 - 176x + 81$

F $\quad x^2 + 196x + 81$

10   The following sequence of transformations is applied to the curve $y = 4x^2$

1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

2. Reflection in the x-axis

3. Stretch parallel to the x-axis with scale factor 2

What is the equation of the resulting curve?

A $y = -x^2 + 12x - 31$

B $y = -x^2 + 12x - 41$

C $y = x^2 + 12x + 31$

D $y = x^2 + 12x + 41$

E $y = -16x^2 + 48x - 31$

F $y = -16x^2 + 48x - 41$

G $y = 16x^2 - 48x + 31$

H $y = 16x^2 - 48x + 41$

11 The quadratic function shown passes through $(2,0)$ and $(q,0)$, where $q > 2$.
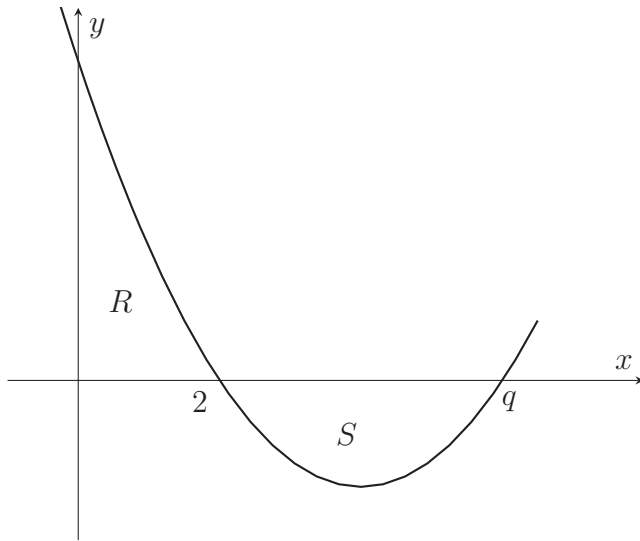
What is the value of q such that the area of region R equals the area of region S?

A $\sqrt{6}$

B 3

C $\frac{18}{5}$

D 4

E 6

F $\frac{33}{5}$

12   How many real solutions are there to the equation

$$3\cos x = \sqrt{x}$$

where x is in radians?

A 0

B 1

C 2

D 3

E 4

F 5

G infinitely many

13 Find the coefficient of x^2y^4 in the expansion of $(1 + x + y^2)^7$

    A   6

    B   10

    C   21

    D   35

    E   105

    F   210

14 The area enclosed between the line $y = mx$ and the curve $y = x^3$ is 6.

What is the value of m?

A 2

B 4

C $\sqrt{3}$

D $\sqrt{6}$

E $2\sqrt{3}$

F $2\sqrt{6}$

15 Find the positive difference between the two real values of x for which

$$\left(\log_2 x\right)^4 + 12\left(\log_2\left(\frac{1}{x}\right)\right)^2 - 2^6 = 0$$

A 4

B 16

C $\frac{15}{4}$

D $\frac{17}{4}$

E $\frac{255}{16}$

F $\frac{257}{16}$

16 The circle C_1 has equation $(x + 2)^2 + (y - 1)^2 = 3$

The circle C_2 has equation $(x - 4)^2 + (y - 1)^2 = 3$

The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x-axis is θ

Find the value of $\tan\theta$

A $\frac{1}{2}$

B 2

C $\frac{\sqrt{2}}{2}$

D $\sqrt{2}$

E $\frac{\sqrt{6}}{2}$

F $\frac{\sqrt{6}}{3}$

G $\frac{\sqrt{3}}{3}$

H $\sqrt{3}$

17 Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.

A $0 < m < \frac{1}{4c}$

B $0 < m < 4c^2$

C $m > \frac{1}{4c}$

D $m < \frac{1}{4c}$

E $m > 4c^2$

F $m < 4c^2$

18 Find the number of solutions and the sum of the solutions of the equation

$$1 - 2\cos^2 x = |\cos x|$$

where $0 \le x \le 180°$

A Number of solutions $= 2$ Sum of solutions $= 180°$

B Number of solutions $= 2$ Sum of solutions $= 240°$

C Number of solutions $= 3$ Sum of solutions $= 180°$

D Number of solutions $= 3$ Sum of solutions $= 360°$

E Number of solutions $= 4$ Sum of solutions $= 240°$

F Number of solutions $= 4$ Sum of solutions $= 360°$

19 Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

    A 26

    B 27

    C 51

    D 52

    E 53

    F 54

20   For how many values of a is the equation

$$(x - a)(x^2 - x + a) = 0$$

satisfied by exactly two distinct values of x?

A   0

B   1

C   2

D   3

E   4

F   more than 4

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