Cambridge Assessment Admissions Testing

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

PAPER 2 **D513/02**

2023 **75 minutes**

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Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one.
You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

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1 Given that

$$\frac{1}{\sqrt{x}-6}-\frac{1}{\sqrt{x}+6}=\frac{3}{11}$$

what is the value of x?

A $2\sqrt{15}$

B $4\sqrt{5}$

C $5\sqrt{2}$

D $\sqrt{58}$

E 50

F 58

G 60

H 80

2 Evaluate

$$\int_9^{16}\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)^2 \mathrm{d}x \;-\; \int_9^{16}\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)^2 \mathrm{d}x$$

A 0

B 2

C 4

D 7

E 14

F 28

G 75

H 175

3 Consider the claim:

For all positive real numbers x and y,

$$\sqrt{x^y} = x^{\sqrt{y}}$$

Which of the following is/are a **counterexample** to the claim?

I $x = 1,\ y = 16$

II $x = 2,\ y = 8$

III $x = 3,\ y = 4$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

4 A student attempts to answer the following question.

What is the largest number of consecutive odd integers that are all prime?

The student's attempt is as follows:

I There are two consecutive odd integers that are prime (for example: 17, 19).

II Any three consecutive odd integers can be written in the form $n - 2$, n, $n + 2$ for some n.

III If n is one more than a multiple of 3, then $n + 2$ is a multiple of 3.

IV If n is two more than a multiple of 3, then $n - 2$ is a multiple of 3.

V The only other possibility is that n is a multiple of 3.

VI In each case, one of the integers is a multiple of 3, so not prime.

VII Therefore the largest number of consecutive odd integers that are all prime is two.

Which of the following best describes this attempt?

A It is completely correct.

B It is incorrect, and the first error is on line I.

C It is incorrect, and the first error is on line II.

D It is incorrect, and the first error is on line III.

E It is incorrect, and the first error is on line IV.

F It is incorrect, and the first error is on line V.

G It is incorrect, and the first error is on line VI.

H It is incorrect, and the first error is on line VII.

5 Consider the two statements

R: k is an integer multiple of π

S: $\int_0^k \sin 2x \, \mathrm{d}x = 0$

Which of the following statements is true?

A R is **necessary and sufficient** for S.

B R is **necessary** but **not sufficient** for S.

C R is **sufficient** but **not necessary** for S.

D R is **not necessary** and **not sufficient** for S.

6 Consider the following equation where a is a real number and $a > 1$:

$$(*) \quad a^x = x$$

Which of the following equations must have the same number of real solutions as $(*)$?

I $\log_a x = x$

II $a^{2x} = x^2$

III $a^{2x} = 2x$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

7 The graph of the line $ax + by = c$ is drawn, where a, b and c are real non-zero constants.

Which one of the following is a **necessary** but **not sufficient** condition for the line to have a positive gradient **and** a positive y-intercept?

A $\frac{c}{b} > 0$ and $\frac{a}{b} < 0$

B $\frac{c}{b} < 0$ and $\frac{a}{b} > 0$

C $a > b > c$

D $a < b < c$

E a and c have opposite signs

F a and c have the same sign

8 A student draws a triangle that is acute-angled or obtuse-angled but **not** right-angled.

The student counts the number of straight lines that divide the triangle into two triangles, **at least one** of which is right-angled.

Which of the following statements is/are true?

I The student can draw a triangle for which there is exactly 1 such straight line.

II The student can draw a triangle for which there are exactly 2 such straight lines.

III The student can draw a triangle for which there are exactly 3 such straight lines.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

9 Consider the following statement about a pentagon P:

(∗) **If** at least one of the interior angles in P is 108°, **then** all the interior angles in P form an arithmetic sequence.

Which of the following is/are true?

I The statement (∗)

II The contrapositive of (∗)

III The converse of (∗)

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

10 Here is an attempt to solve the inequality $x^4 - 2x^2 - 3 < 0$ by completing the square:

$x^4 - 2x^2 - 3 < 0$

I if and only if $x^4 - 2x^2 + 1 < 4$

II if and only if $(x^2 - 1)^2 < 4$

III if and only if $-2 < x^2 - 1 < 2$

IV if and only if $x^2 - 1 < 2$

V if and only if $x^2 < 3$

VI if and only if $-\sqrt{3} < x < \sqrt{3}$

Which of the following statements is true?

A The argument is completely correct.

B The first error occurs in line I.

C The first error occurs in line II.

D The first error occurs in line III.

E The first error occurs in line IV.

F The first error occurs in line V.

G The first error occurs in line VI.

11 In this question, k is a positive integer.

Consider the following theorem:

If $2^k + 1$ is a prime, **then** k is a power of 2. $(*)$

Which of the following statements, taken individually, is/are equivalent to $(*)$?

I **If** k is a power of 2, **then** $2^k + 1$ is prime.

II $2^k + 1$ is **not** prime **only if** k is **not** a power of 2.

III A **sufficient** condition for k to be a power of 2 is that $2^k + 1$ is prime.

	Statement I is equivalent to $(*)$	Statement II is equivalent to $(*)$	Statement III is equivalent to $(*)$
A	Yes	Yes	Yes
B	Yes	Yes	No
C	Yes	No	Yes
D	Yes	No	No
E	No	Yes	Yes
F	No	Yes	No
G	No	No	Yes
H	No	No	No

12 In this question, p is a real constant.

The equation $\sin x \cos^2 x = p^2 \sin x$ has n distinct solutions in the range $0 \leq x \leq 2\pi$

Which of the following statements is/are true?

I $n = 3$ is **sufficient** for $p > 1$

II $n = 7$ **only if** $-1 < p < 1$

A none of them

B I only

C II only

D I and II

13 Let x be a real number.

Which **one** of the following statements is a **sufficient** condition for **exactly** three of the other four statements?

A $x \geq 0$

B $x = 1$

C $x = 0$ **or** $x = 1$

D $x \geq 0$ **or** $x \leq 1$

E $x \geq 0$ **and** $x \leq 1$

14 Three lines are given by the equations:

$$ax + by + c = 0$$

$$bx + cy + a = 0$$

$$cx + ay + b = 0$$

where a, b and c are non-zero real numbers.

Which one of the following is correct?

A **If** two of the lines are parallel, **then** all three are parallel.

B **If** two of the lines are parallel, **then** the third is perpendicular to the other two.

C **If** two of the lines are parallel, **then** the third is parallel to $y = x$.

D **If** two of the lines are parallel, **then** the third is perpendicular to $y = x$.

E **If** two of the lines are perpendicular, **then** all three meet at a point.

F **If** two of the lines are perpendicular, **then** the third is parallel to $y = x$.

G **If** two of the lines are perpendicular, **then** the third is perpendicular to $y = x$.

15 The base 10 number 0.03841 has the value

$$0 \times 10^{-1} + 3 \times 10^{-2} + 8 \times 10^{-3} + 4 \times 10^{-4} + 1 \times 10^{-5} = 0.03841$$

Similarly, the base 2 number 0.01101 has the value

$$0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5} = \frac{13}{32}$$

What is the value of the recurring base 2 number $0.\dot{0}0\dot{1}1\dot{} = 0.001100110011\ldots$?

A $\frac{1}{3}$

B $\frac{1}{5}$

C $\frac{1}{15}$

D $\frac{2}{15}$

E $\frac{4}{15}$

F $\frac{3}{16}$

G $\frac{5}{16}$

H $\frac{6}{31}$

16 A sequence is defined by:

$$u_1 = a$$
$$u_2 = b$$
$$u_{n+2} = u_n + u_{n+1} \quad \text{for } n \geq 1$$

where a and b are positive integers. The highest common factor of a and b is 7.

Which of the following statements **must** be true?

I u_{2023} is a multiple of 7

II If u_1 is not a factor of u_2, then u_1 is not a factor of u_n for any $n > 1$

III The highest common factor of u_1 and u_5 is 7

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

17 The ceiling of x, written $\lceil x \rceil$, is defined to be the value of x rounded up to the nearest integer.

For example: $\lceil \pi \rceil = 4$, $\lceil 2.1 \rceil = 3$, $\lceil 8 \rceil = 8$

What is the value of the following integral?

$$\int_0^{99} 2^{\lceil x \rceil} \, dx$$

A 2^{99}

B $2^{99} - 1$

C $2^{99} - 2$

D 2^{100}

E $2^{100} - 1$

F $2^{100} - 2$

18 The equation $x^4 + bx^2 + c = 0$ has four distinct real roots **if and only if** which of the following conditions is satisfied?

A $b^2 > 4c$

B $b^2 < 4c$

C $c > 0$ **and** $b > 2\sqrt{c}$

D $c > 0$ **and** $b < -2\sqrt{c}$

E $c < 0$ **and** $b < 0$

F $c < 0$ **and** $b > 0$

19 In this question, $f(x)$ is a non-constant polynomial, and $g(x) = xf'(x)$

$f(x) = 0$ for exactly M real values of x.

$g(x) = 0$ for exactly N real values of x.

Which of the following statements is/are true?

I It is possible that $M < N$

II It is possible that $M = N$

III It is possible that $M > N$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

20 Let f be a polynomial with real coefficients.

The integral $I_{p,q}$ where $p < q$ is defined by

$$I_{p,q} = \int_{p}^{q} (f(x))^2 - (f(|x|))^2 \, \mathrm{d}x$$

Which of the following statements must be true?

1 $I_{p,q} = 0$ **only if** $0 < p$

2 $f'(x) < 0$ for all x **only if** $I_{p,q} < 0$ for all $p < q < 0$

3 $I_{p,q} > 0$ **only if** $p < 0$

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

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